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Reliability of methods to separate stress tensors from heterogeneous fault-slip data

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Abstract

The reliability of methods for separating palaeostress tensors from heterogeneous fault-slip data is evaluated. The methods of Etchecopar et al. (1981), Yamaji (2000), and the cluster procedure of Nemcok and Lisle (1995) are assessed but the results can probably be extrapolated to other methods based on similar assumptions. Heterogeneous fault-slip data sets, artificially generated by mixing two natural homogeneous data sets, have been used to evaluate both the role of the relative dominance (in number of faults taken from each tensor) and the difference between the parent tensors. The results obtained from a natural heterogeneous data set were compared with additional field data to evaluate and constrain the tensor separation process as well. Results suggest that attempts to devise a fully automatic separation procedure for distinguishing homogeneous data sets from heterogeneous ones will be unsuccessful because the researcher will always be required to take some part in the correct choice of the tensors. In this sense, additional structural data such as geometrical characteristics of the faults (e.g. conjugate or quasi-conjugate Andersonian systems), stylolites or tension gashes will be very useful for the correct separation of stress tensors from fault-slip data.

Keywords: Heterogeneous fault-slip data; Fault analysis; Stress tensor separation

1. Introduction

Since the 1970s a variety of methods have been proposed for estimating palaeostress states from field measurements of fault striations on fault planes (e.g. Carey and Brunier, 1974; Angelier and Mechler, 1977; Angelier, 1979a, 1984; Etchecopar et al., 1981; Simón-Gómez, 1986; Lisle, 1987, 1988; Hardcastle and Hills, 1991). The majority of these methods assume that the sampled faults slipped independently in a homogeneous stress field and that the recorded fault-slip represents the direction of maximum shear stress on the fault plane. Evidence such as the reactivation of fault planes, faults with similar orientations but opposite senses of slip, or several sets of stylolites and tension gashes are common in nature but are not compatible with a single stress tensor. Fault-slip data that can only be explained by more than one stress tensor have been commonly called heterogeneous.

Accordingly, several methods have been developed for separating stresses from heterogeneous fault-slip data, e.g. Angelier (1979a), Etchecopar et al. (1981), Armijo et al. (1982), Huang (1988) and Galindo-Zaldívar and González-Lodeiro (1988). A detailed analysis of most of these methods was presented by Angelier (1994). Recently, new methods, such as that based on cluster analysis (Nemcok and Lisle, 1995), the graphical procedure of Fry (1999), or the multiple inverse method (Yamaji, 2000), have been proposed for analysing heterogeneous fault-slip data sets. There is an attempt to propose automatic computer-based methods where no decisions need to be taken by the structural geologist during the analysis. This goal is considered essential for all researchers dealing with interpretation of heterogeneous fault slip data. Even though a number of different methods have been proposed, there is no study that analyses the relative reliability of results from the various methods. These methods are based on different assumptions and use very different approximations both to analyse faults and to separate stress tensors, and could therefore potentially yield different results.

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The methods for analysing and separating stress tensors from heterogeneous fault-slip data can be roughly grouped into three essential procedures: manual procedures, semi-automatic procedures that minimize a parameter, and automatic procedures based on attributes of faults. The manual procedures are based on graphical representations of the results, which are used to differentiate stress tensors. Some of the graphical procedures, such as the Right Dihedra (Angelier and Mechler, 1977) and Right Trihedra (Lisle, 1987, 1988) methods, allow the recognition that data are heterogeneous, but do not allow the component tensors to be identified. On the other hand, the graphical y–R diagram method (Simón-Gómez, 1986) and its modification made by Fry (1992) as well as the methods of Fry (1999) and Yamaji (2000) permit the separation of tensors from the analysis of clouds and clusters displayed in them.

The second category, semiautomatic procedures, comprises numerical methods that automatically minimize some parameter, usually the sum of the angular misfits (differences between theoretical striae predicted from some trial tensor and the real striae), to search for the best tensor to fit the faults and then analyse the remaining non-fitting faults to search for other tensors. Although these inversion methods are automatic, the separation of different stress tensors and therefore homogeneous fault data need to be carried out manually, so that they can be considered as semiautomatic procedures for separating stress tensors. Methods that use this approach are those proposed by Etchecopar et al. (1981), Armijo et al. (1982), Angelier (1979a, 1984), and Galindo-Zaldívar and González-Lodeiro (1988) amongst others.

In the third category, automatic separation of homogeneous data sets from heterogeneous data, the fault slip data are separated before the inversion process takes place, e.g. the cluster separation procedure proposed by Nemcok and Lisle (1995). In this method, the striation data of each fault are checked for compatibility with a large number of tensors. Each fault is then described by a large quantity of attributes that characterize the fault's fit to each of the tensors. These attributes are later used for comparing and grouping faults.

Most of the new techniques have been tested with theoretical fault-slip data sets made up of fault orientations usually randomly distributed in space, and striations on the faults calculated according to the Bott equation (i.e. the direct procedure). However, natural faults commonly display several well-formed sets. Thus, conjugate or quasi-conjugate fault systems are common in nature (Angelier, 1979b). In addition, when natural heterogeneous fault-slip data sets are used in the tests the results are usually satisfactory often because the two individual tensors are quite different.

This paper addresses the feasibility of separating stresses from heterogeneous fault-slip data and deals with the reliability of results obtained from different procedures and with different input conditions as well as the feasibility of the automatic separation of stress tensors. Different tests have been performed with both natural and artificial heterogeneous data sets by using three different methods (the Etchecopar, Yamaji and cluster methods) for separating tensors. The first test utilises artificial heterogeneous data created by mixing two homogeneous natural data sets. In this test, we evaluate the effect of the relative dominance of each individual data set on the separation process. In the second test, the role of the relative proximity in orientation of both individual stress tensors in the separation procedure has been explored. In the third test, natural heterogeneous data are analysed by the chosen methods, and their results are contrasted with other independent structural information.

2. Methods used in this study

The methods are the numerical method of Etchecopar et al. (1981), the multiple inversion method of Yamaji (2000) and the cluster analysis procedure of Nemcok and Lisle (1995). This choice is based on the fact that these represent the three main ways proposed in the literature for separating stress tensors from heterogeneous data. We refer to these as the Etchecopar method, the Yamaji method, and the cluster analysis method, respectively.

The Etchecopar method (Etchecopar et al., 1981; Etchecopar, 1984) was used in combination with the Right Dihedra method (Angelier and Mechler, 1977) and the y–R diagram method (Simón-Gómez, 1986) in the manner proposed by Casas et al. (1990). This combined use provides a more robust result, and contrasting their individual results allow us to assess the reliability of results. The Etchecopar method uses an iterative algorithm that minimizes the quadratic sum S of the angular deviations choosing a selected percentage (n) of data that have smallest angles between the theoretical and actual striations. The final percentage of faults (n) used for determining the stress tensor (Etchecopar et al., 1981) is the one which produces a maximum number of striations giving small deviation, gives a histogram of angle of deviations for which the maximum corresponds to the smallest differences in angle, and leads to stable solutions. We have used a misfit threshold of 12° in the general case and 10° for the case of tensors with a strike-slip configuration following the recommendation by Casas (1990) and Casas et al. (1990). This method yields: (1) the orientation of principal stress axes, (2) the shape factor R_e ($R_e = \sigma_2 - \sigma_3/\sigma_1 - \sigma_3$) of the stress ellipsoid, (3) the estimated errors for the calculated stress axis orientations and the R_e parameter, (4) the histogram of angular deviations of faults, and (5) the representation on a Mohr diagram of the fault planes explained by the computed tensor. These results together provide a complementary way of evaluating the quality of the solution (Etchecopar et al., 1981; Etchecopar, 1984; Etchecopar and Mattauer, 1988; Casas, 1990; Casas et al., 1990; Liesa, 2000). As here defined, the R_e parameter is equal to the shape factor Φ (Angelier, 1994).

The general procedure used here for the application of the Etchecopar method involves the following steps. Firstly, the program searches for tensors with a high percentage of explained faults (80% for example). This percentage is increased or reduced until the quality criteria (mainly the misfit threshold and shape of the angular deviation histogram) suggest that it is a good result. Afterwards, a minor second tensor is sought in the same way. Sometimes, usually when the number of faults associated with the second tensor is small, just the data that are not explained by the first computed tensor must be used for obtaining the other tensor. Occasionally, obtaining a good solution requires the input of an initial tensor that becomes the starting point for the search algorithm. Thus, besides the percentage, input parameters include the orientations of the σ_1 and σ_3 stress axes and the R_e value. They are selected on the basis of results of other fault analysis methods that permit the exploration of all the space of possible solutions (y–R diagram, for example) or information provided by other tectonic structures (such as stylolites). The use of the Etchecopar method in combination with other methods overcomes the problem of 'hidden' tensors. In any case, the selection of appropriate tensors must be made using additional criteria such as the mechanical compatibility or the tectonic context (Casas et al., 1990).

The statistical technique of cluster analysis arranges faults into dynamic groups. The grouping procedure is based on attributes that describe a given fault's compatibility with a range of trial stress tensors. The background and procedure of the cluster analysis are fully explained by Nemcok and Lisle (1995) and Lisle and Vandycke (1996). A modified version of the cluster program was used in this study (Appendix A). The subgroups of faults obtained after the application of the cluster method were analysed by the Etchecopar method in order to calculate the stress tensors.

The multiple inverse method (Yamaji, 2000) applies the inverse method for determining the directions of the three principal axes and the shape of stress ellipsoid for all k-fault subsets of the data. After an evaluation of the stability of the solution, an optimal k-value (usually $k = 4$ or 5) is chosen and significant solutions are identified as clusters in parameter space.

3. Testing the simulated heterogeneous fault-slip data

3.1. Data

In order to obtain a heterogeneous fault slip data set, two natural homogeneous fault data sets were mixed. The first set (site TYM) consists of 38 normal faults (Fig. 1) measured by Angelier (1979b) to the SSE of Tymbaki (western Messara, Crete). The second data set (site ITUR) consists of 35 strike-slip faults (Fig. 1) selected from all measurements made on Middle Eocene calcareous sandstones to the north of Estella (Navarra, Spain). This data set taken from Liesa (2000) was not homogeneous in character, so that several normal faults explained by a different stress tensor were removed from the original data set. In the original data, the cross-cutting relationships of striations on several fault planes indicate that the normal movement acted later than the strike-slip one (Liesa, 2000). This suggests that our artificial separation identifies a homogeneous fault data set belonging to the strike-slip tensor configuration that acted prior to the extensional event. In both fault data sets all senses of movements can be clearly deduced from kinematic indicators (striated or crystallized steps), so that this is not an additional area of uncertainty in the data. The tensors obtained from the inversion of each individual data set are displayed in Fig. 1 and Table 1.

3.2. The relative dominance of subgroups

For testing the influence of the relative dominance of subgroups on the separation of homogeneous fault populations, two different heterogeneous data sets are investigated. In the first one (case 1), the data represent the combination of all fault-slip data of both natural subsets (TYM and ITUR sites). In this case, there is no dominance between both individual data groups, because they consist of 38 faults (TYM subgroup) and 35 faults (ITUR subgroup), respectively. In the second (case 2), an artificial heterogeneous fault data is created by adding the TYM subgroup to one third of the faults (12 faults) randomly chosen of the ITUR subgroup. This second case represents a relative dominance of 3:1 between the individual subgroups.

In the first case with a similar relative dominance of the subgroups, the results obtained from the different methods were very similar and very close to the expected stress tensors (Table 2). In the second case with unequal relative dominance of subgroups, the cluster procedure and

Table 1
Expected stress tensors from the individual TYM and ITUR homogeneous subgroups of fault-slip data. The results have been calculated using the Etchecopar (Etchecopar et al., 1981) inversion method. The value of α is the average deviation angle (in grades) is (in degrees) between theoretical and real striae and N is the number of faults explained by the tensor (the total number of faults separated by the cluster procedure is shown between brackets)

	σ_1 axis	σ_3 axis	$\Phi = R_e$	α	N
	Original data				
TYM subgroup	278 81	144 06	0.06	14.4	38
TYM subgroup[a]	278 84	140 06	0.08	12.1	35
ITUR subgroup	227 00	137 04	0.23	7.7	35
	Modified (rotated) TYM subgroup				
Intermediate TYM data	056 04	144 04	0.06	14.4	38
TYM data in case 4	076 04	164 04	0.06	14.4	38
TYM data in case 3	106 04	194 04	0.06	14.4	38

[a] Tensor used for abating the faults to a strike-slip configuration.

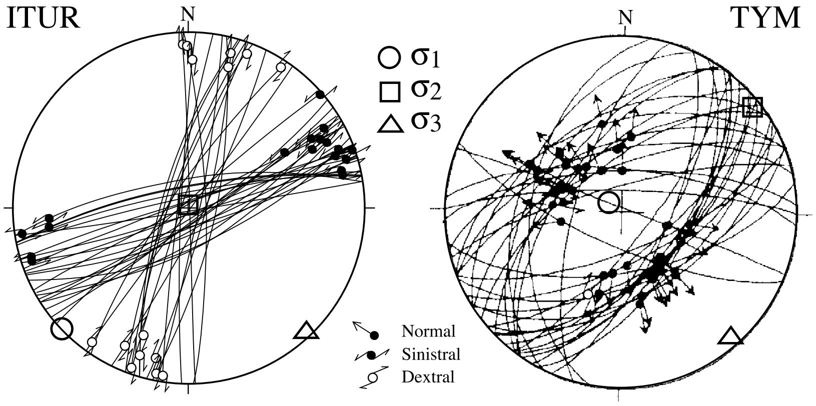


Fig. 1. Stereographic projection (lower hemisphere, Schmidt net) of faults and striae of the homogeneous fault slip data of ITUR and TYM sites and of the stress axes of stress tensor (see Table 1) determined from them.

Etchecopar method successfully separated the two stress tensors, whereas the Yamaji method was not able to determine the minor stress tensor (Fig. 2a). With this exception, the obtained tensors in cases 1 and 2 are in good agreement with those obtained from the individual data subsets (Fig. 1; Table 1).

3.2.1. Interpretation

The analysis that follows demonstrates that the most significant factor determining the unsuccessful results of the Yamaji method in case 2 (relative dominance 3:1) is the relative size of fault sets and not the small number of faults (12) of the minor fault data set, near the minimum required to define the stress tensor with confidence. The failure of the Yamaji method to determine the minor tensor suggests an important drawback of this method. Its cause can be found in the analysis procedure, which leads to a very different probability in the number of subsets and therefore in the solutions when the number of faults of each subset is quite different. To explain this assumption let us consider, as is the case, a heterogeneous data set formed by two homogeneous data sets with 38 and 12 data, respectively. According to the Yamaji (2000) formula that calculates the number of subsets C of k-elements:

$$_{N}C_{k} = N!/k(N-k)!$$

where N is the total number of fault-slip data and k the number of faults in each subset used for inversion, and for a value of $k = 5$ (which is considered a normal value where solutions are stable; Yamaji, 2000), the total number C of subsets of five faults will be 2,118,760, while the number of k-subsets that incorporate exclusively faults of the first (38 faults) or second (12) homogeneous fault data set will be 501,942 and 792 subsets, respectively. Not taking into account the combinations between faults of different homogeneous fault sets, which would increase the value, the ratio between subsets that will fit the first tensor to those that fit the second tensor is 634. So, with a relative dominance of 3:1 in the number of faults between both homogeneous subgroups a relative dominance greater than 634:1 is expected in results that fit the first or the second tensor, respectively. This fact explains why the minor solution is very difficult to detect by the Yamaji method. Therefore, such a method probably only achieves satisfactory solutions when data of homogenous subsets that make up a heterogeneous fault-slip data set roughly have equal dominance.

Finally, the Yamaji method has the problem of determining the shape parameter of the stress tensor because the cluster of results involves a wide range of μ_L values (Fig. 2a). Additional drawbacks are that this method permits neither the classification of faults into subsets nor the determination of the number of faults for each solution tensor, allowing us just to recognise the stress tensors. Moreover, the results must be evaluated by visual inspection, which carries the potential for bias.

The cluster analysis method performed best in distinguishing between individual subgroups in the case of subsets with variable dominance. So, all faults were usually

Table 2

Testing the influence of the relative dominance of individual subgroups in the separation of stress tensors. Stress tensors (σ_1 and σ_3 stress axis orientations and Φ parameter) obtained by the different used procedures for no relative dominance between both individual subsets (case 1) and for an unequal (3:1) relative dominance of subgroups of faults (case 2). Key as Table 1

	σ_1 axis	σ_3 axis	$\Phi = R_e$	α	N
Case 1: Nearly equal relative dominance of subgroups in number of faults					
Etchecopar method	209 85	320 02	0.20	13.4	42
	041 07	131 04	0.23	7.5	37
Yamaji method	218 88	131 04	0.12	–	–
	044 02	134 03	0.20	–	–
Cluster procedure	278 81	144 06	0.06	14.4	38(38)
	227 00	137 04	0.23	7.7	35(35)
Case 2: Unequal (3:1) relative dominance of subgroups in number of faults					
Etchecopar method	176 82	328 07	0.26	13.2	39
	045 02	135 13	0.14	4.5	12
Yamaji method	240 86	316 14	0.1–0.4	–	–
Cluster procedure	278 81	142 07	0.07	14.4	37(37)
	045 02	135 13	0.14	4.5	12(12)

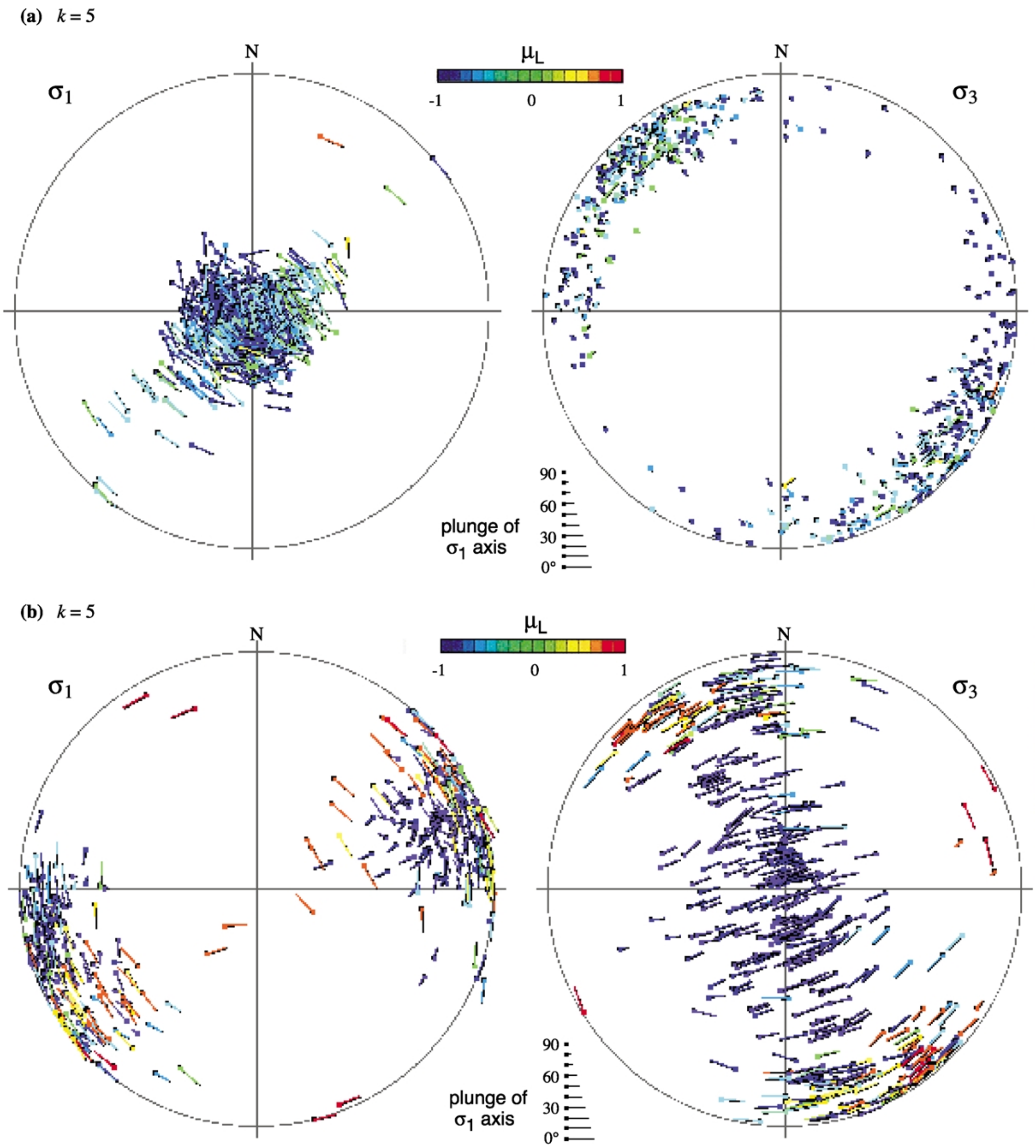


Fig. 2. Results of the multiple inverse method of Yamaji (2000). (a) Case 2: an unequal relative dominance of the individual subgroups (enhance factor $E = 4$; dispersion factor $D = 2$; see Yamaji (2000) for explanation). (b) Case 3: σ_2 vertical and approximate 60° difference between the σ_1 directions of both expected individual tensors (enhance factor $E = 6$; dispersion factor $D = 3$). The parameter μ_L (Lode's number) represents the shape of the stress ellipsoid and is defined (Lode, 1925) by the ratio $\mu_L = (2\sigma_2 - \sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)$; where σ_1, σ_2 and σ_3 are the principal stresses. The parameter μ_L is linearly related to ϕ or R_e parameters according to $\mu_L = 2R_e - 1$.

grouped in the correct subgroup for case 1 as well as for case 2. Only in some tests of case 2 was one strike-slip fault grouped with the normal fault (TYM) subgroup.

Although the results obtained from the Etchecopar method in case 1 are in good agreement with the general stress tensor determination, four or five faults were misclassified and explained by the other tensor. This may be caused by the fact that both expected individual tensors are very close in axial orientations, although σ_1 and σ_2 have interchanged positions. In any case, the position of these faults on the $\sigma_2\sigma_3$ circle on the Mohr circle diagram presented by this method suggest that these faults are not well explained by the tensor, and this factor has been used to remove them from the solution. A second tensor was then searched for considering those faults not consistent with the first tensor and those faults not favourably oriented for slip. Including these faults allows us to achieve a better solution for the second tensor. In addition, the determination for case 2 of the minor stress tensor by the Etchecopar method is more difficult because the small number of faults (12 faults) that characterizes the tensor leads to a poor tensor definition.

3.3. The difference in tensors from individual subgroups

The influence of the similarity in orientation of both individual stress tensors in the separation procedure was evaluated using data sets TYM and ITUR as well. In this case, two different tests were carried out with an angular difference, in the horizontal plane, of the σ_1 orientations of the parent stress tensors of 60° (case 3) and 30° (case 4), respectively, σ_2 being nearly vertical (strike-slip configuration) in both cases. For producing such conditions, normal fault data of set TYM were firstly restored by rotating the $\sigma_1\sigma_3$ plane of the initial stress tensor (Table 1) to the horizontal, so that σ_1 was horizontal and normal faults were converted to strike-slip faults. Then, these faults were clockwise rotated 50° and 20° about a vertical axis to obtain roughly the angular deviation of 60° (case 3) and 30° (case 4), respectively, between the expected σ_1 with the new TYM data and the σ_1 of the stress tensor obtained from ITUR data. Finally, the modified TYM data were merged with ITUR data to provide the required heterogeneous fault-slip data sets.

In case 3, with σ_2 vertical and a roughly 60° difference between the σ_1 directions of both expected individual tensors or parent tensors (Table 1, case 3), the cluster and Etchecopar methods separate the individual tensors successfully. The Yamaji method, however, is unsuccessful for this case and does not distinguish both tensors, so that just one tensor is obtained, which displayed a σ_1 axis trending 073°, i.e. in an intermediate position between those expected for the two individual subgroups (Table 3; Fig. 2b).

In case 4 (with σ_2 vertical and an approximate 30° difference between σ_1 of expected individual tensors (Table 1, case 4)), all methods failed to separate the tensors. In this case, all faults are found to be compatible with a single tensor with σ_1 orientation intermediate to those of the expected tensors.

3.3.1. Interpretation

Based on results in case 4, it seems that for these methods, the 30° difference angle between tensors would be a limiting angle for the separation of stress tensors. In such cases, field observations, such as similarly oriented fault planes with opposite sense of slip or several striae on a fault plane, could suggest the existence of more than one stress tensor although computer methods only provide a compromise stress tensor.

The failure of the Yamaji method in case 3 could also be a consequence of combining faults to form subsets of k elements that are used for stress inversion. In this way, the number of combinations involving faults of different subgroups will be greater than the combinations involving faults from same subgroup of faults, which leads to the major abundance of intermediate solutions.

The cluster procedure usually achieves a stable separation of the subgroups in case 3, but in a test with other input conditions (grid spacing = 15°, misfit threshold = 10°, and no shear (τ/σ stress ratio) condition) a different cluster pattern with incorrect subsets appeared. As in all the cases, the use of a high value for the misfit threshold between real and theoretical striations also leads to incorrect results. Besides, also in the successful tests with different input parameters (grid spacing = 30°, 30°, and 20°; misfit threshold = 10°, 10°, and 10°; and τ/σ stress ratio 0.7, non-considered, and 0.7, respectively) three faults were placed in the wrong subgroup. In any case, three faults from nearly 35 faults should not produce significant errors in the final solution of the stress tensor especially when these faults are very similar to the others.

The Etchecopar method infers two tensors very close to those expected (Table 3). These tensors explain the majority of faults of the respective subgroups of data. However, as in cases 1 and 2, three faults of the TYM subgroup of data (faults 23, 27 and 28) were not properly explained and four faults (faults 39, 43, 50 and 61) of the other (ITUR) subgroup were explained by the first tensor. The second tensor explained all faults of the subgroup ITUR and three

Table 3
Testing the influence of the difference in stress tensors of the individual subgroups in the separation of stress tensors. Stress tensors (σ_1 and σ_3 stress axis orientations and Φ parameter) obtained by the different used procedures for 60° of angular difference in the horizontal plane between the σ_1 orientations of the expected individual tensors (case 3) and for a difference of 30° (case 4). Key as Table 1

		σ_1 axis	σ_3 axis	$\Phi = R_e$	α	N
Case 3: Separation between individual tensors of 60°						
Etchecopar method	First	096 00	006 02	0.17	12.0	39
	Second	229 00	139 04	0.21	7.9	38
Yamaji method		073 01	163 06[a]	0.9	–	–
Cluster procedure		104 01	014 03	0.06	12.0	39(41)
		047 00	137 04	0.24	7.6	32(32)
Case 4: Separation between individual tensors of 30°						
Etchecopar method		055 02	145 10	0.03	11.6	66
Yamaji method		060 03	Great circle	0.1–0.3	–	–
Cluster procedure		055 02	145 10	0.03	11.6	66

[a] A different clustering of σ_3 in a vertical orientation and Φ values of 0.9 is also depicted.

faults (faults 4, 5 and 15) of the subgroup TYM, which also were explained by the first tensor. These three faults are located, however, in the lower-right position on the Mohr circle of the second tensor and are therefore incompatible with it, so that they could be removed from this solution (Fig. 3).

4. Testing a natural heterogeneous data set: comparison with other structural data

4.1. Data

The natural heterogeneous data set consists of 49 fault-slip data pairs measured on steeply dipping strike-slip faults measured on the exposures of Triassic breccias at Ogmore-by-Sea (South Wales). These data (Ogmore site 4) have been previously analysed by Vandycke et al. (1992), Liesa (1993), and Lisle and Vandycke (1996). The choice of this data set was based on the fact that these studies used different inversion methods, which sometimes led to different stress tensor separations (Fig. 4).

Moreover, there is additional structural information that can be used for assessing the validity of the inferred stress tensors and therefore to constrain the results. Lisle and Vandycke (1996) point out that the faults in the Triassic rocks describe a conjugate strike-slip fault set trending at 020° and 065°. These trends have movement senses indicated by calcite slickenfibres and off-set clasts compatible with a post-Triassic NE–SW direction of σ_1. Cross-cutting relationships of two lineation sets on a strike-slip fault trending 105° shows that the dextral movement was prior to the sinistral one. Within the Triassic and especially within the Carboniferous Limestone, located under the Triassic angular unconformity, other nearly vertical strike-slip faults occur in two trends, averaging 100° and 142°, respectively. These faults interpreted as conjugate Andersonian faults indicate a σ_1 direction plunging gently to 120° and a near-vertical σ_2 axis (Lisle and Vandycke, 1996). In addition, two sets of stylolitic points indicating E–W and N–S σ_1 compression, respectively, and two sets of steeply dipping tension gashes indicating NNE–SSW and ESE–WNW σ_3 extension, respectively, are widely displayed within the Carboniferous limestones (Fig. 4).

4.2. Results

The analysis of fault-slip data at Ogmore site 4 by the Etchecopar method, combined with the Right Dihedra and y–R diagram methods, allowed us to separate two stress

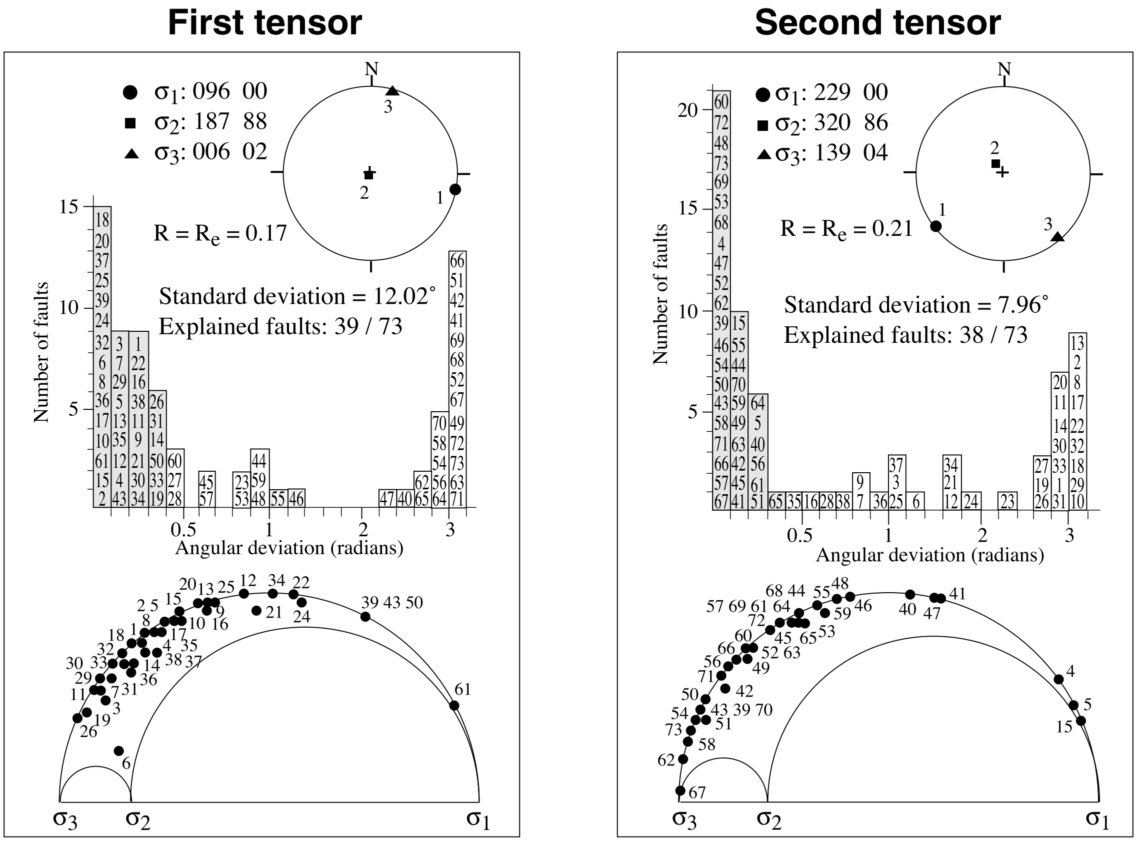
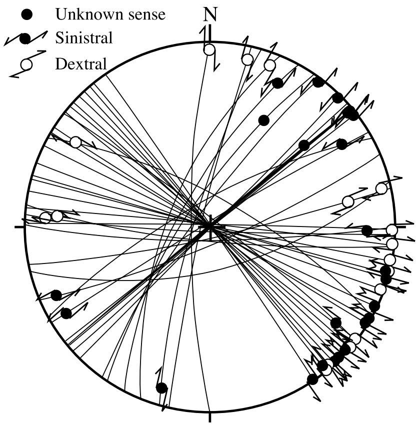


Fig. 3. Two stress tensors obtained by the Etchecopar method (Etchecopar et al., 1981) for case 3. The stress axis orientations, the shape factor (R_e) of stress ellipsoid, the number of explained faults by the tensor, the average standard deviation between the real and theoretical striae of faults explained by the tensor, the histogram of angular deviations for each fault (the explained faults appear in grey), and the Mohr circle representation of the explained faults (faults labelled with their respective numbers) are displayed for each stress tensor. Faults 1–38 are faults of the TYM set; faults 39–73 correspond to faults of the ITUR set.



	Stress tensors	
Liesa (1993)	σ_1: 110 03 σ_3: 020 00 $\phi = 0.24$	σ_1: 034 01 σ_3: 124 08 $\phi = 0.04$
Lisle & Vandycke (1996)	σ_1: 179 08 σ_3: 086 19 $\phi = 0.34$	σ_1: 068 17 σ_3: 160 08 $\phi = 0.9$

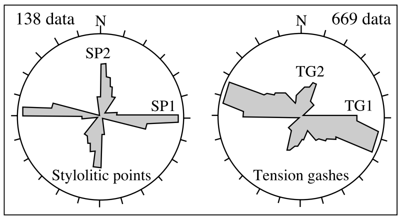


Fig. 4. Strike-slip faults sampled in Ogmore 4 site and stress tensors previously inferred. Rose diagrams of stylolitic points and strikes of tension gashes in Carboniferous Limestone are also shown (after Lisle and Vandycke, 1996).

tensors with strike-slip configurations (Table 4; Fig. 5) explaining 23 and 26 faults, respectively. The first tensor has a maximum principal stress axis σ_1 nearly horizontal and trending 111° and a stress ratio $R_e = 0.24$ ($R_e = \Phi$). The nearly horizontal σ_1 of the second one trends 035° and the stress ratio is 0.17. In this case, three faults (numbered 26, 30 and 34) have been explained by the two tensors but another three faults (numbered 4, 5 and 18) were not explained by either tensor.

The clustering of individual results provided by the Yamaji method also suggests the existence of two distinct stress tensors with approximately horizontal compression directions trending roughly 032° and 307°, respectively (Table 4). Both clusters are depicted by individual solutions representing values of μ_L ranging from -1 to -0.6 and a medium value of -0.8 ($\Phi = 0.1$), and with σ_3 axes distributed on a great circle but with some grouping near the vertical position. Accordingly, the positions of σ_2 and σ_3 axes are not well defined.

These data were intensively investigated by the cluster procedure because earlier results of Lisle and Vandycke (1996) by this method are not in agreement with other field data. So, 18 tests with different input parameters, i.e. different values for grid spacing, misfit threshold, and shear criteria, were carried out (see table on Fig. 6). The results

Table 4
Summary of stress tensors determined in the Ogmore site 4. Keys as Table 1

OGMORE SITE 4	σ_1 axis	σ_3 axis	$\Phi = R_e$	α	N
Etchecopar method	111 03	021 01	0.24	5.91	23
	035 01	125 10	0.17	6.42	26
Cluster procedure	Major cluster agglomeration pattern				
Tensor 3	342 02	072 06	0.48	8.86	25
Tensor 4	068 04	338 06	0.57	5.90	21
	Minor cluster agglomeration pattern				
Tensor 1	101 01	011 01	0.12	2.45	21
Tensor 2	026 01	116 26	0.03	8.18	20
Yamaji method	307 01	Near vertical	0.10	–	–
	032 03	Near vertical	0.10	–	–

suggest two distinct clustering patterns and therefore two different groupings of the fault-slip data that have been called, according to the number of solutions, the major (a) and minor (b) agglomeration patterns (Fig. 6). Each agglomeration pattern produced two main groups of faults, each one of which was inverted by using the Etchecopar method, thus giving two stress tensors for each agglomeration pattern (Table 4; Fig. 6).

4.3. Interpretation

The stress tensors separated from the Ogmore heterogeneous fault-slip data by the Etchecopar and Yamaji methods are very similar and correspond to those inferred from the minor cluster agglomeration pattern obtained by the Nemcok and Lisle (1995) cluster procedure (Table 4). Accordingly, two stress tensors with usually strike-slip configuration (σ_2 nearly vertical) and σ_1 nearly horizontal and oriented roughly ESE–WNW (tensor 1) and NNE–SSW (tensor 2), respectively, can be distinguished. Moreover, the faults grouped in these two homogeneous groups of faults by Etchecopar (Fig. 5) and cluster procedures (Fig. 6b) are also very similar. The similarity of the results obtained by the different methods suggest that these two stress tensors are defined with a high confidence and that they probably represent the inversion of two single groups of homogeneous fault-slip data. However, the appearance of a different solution by cluster analysis, which is moreover the prevalent solution in terms of number of tests (major solution; Fig. 6a; Table 4), is not in agreement with this conclusion. So, the significance of obtaining these two possible solutions by cluster analysis as well as the 'election' process of the right result are two aspects that require closer consideration.

The fault agglomeration patterns and tensors obtained from their inversion by the Etchecopar method, for the major and minor solutions inferred from cluster procedure are shown in Fig. 6. The four tensors are relatively good from the point of view of the quality criteria of the angular

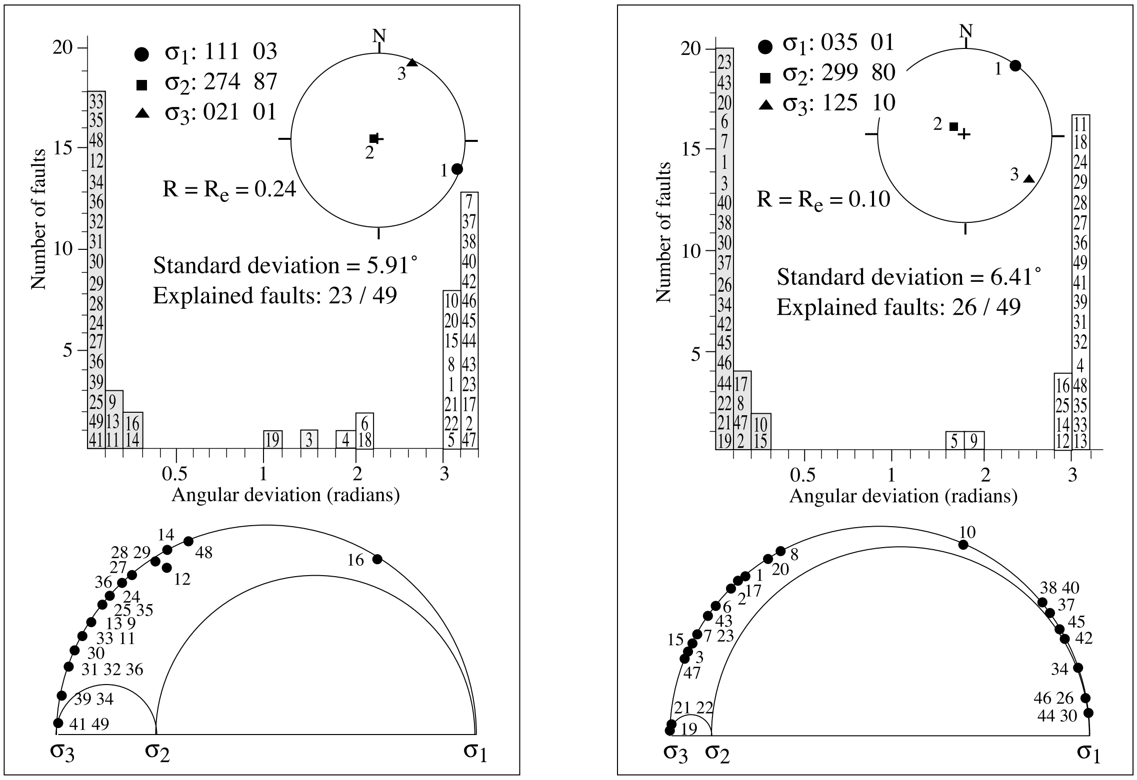


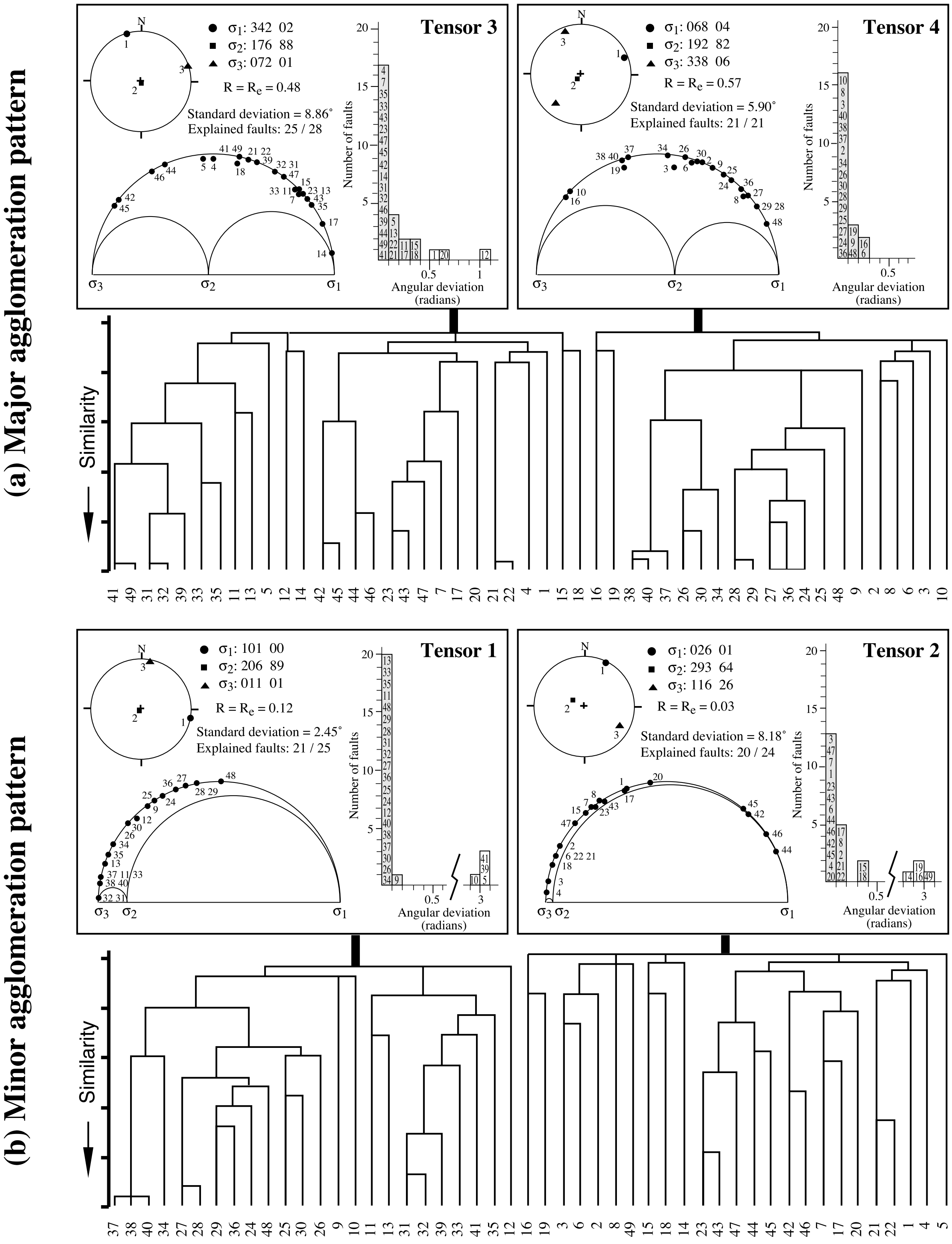
Fig. 5. Stress tensors obtained by Etchecopar method for the Ogmore 4 site fault-slip data. Results are displayed as in Fig. 3.

deviation between real and theoretical striae and of the angular deviation frequency histogram, but they show a significant difference with respect to the location of faults on the Mohr circle (Fig. 6). In tensors 1 and 2 the fault planes are favourably oriented for slip, whereas in tensors 3 and 4 they are not. Accordingly, tensors 1 and 2, those that represent the minor solution of the cluster procedure, are mechanically more consistent with data and must be considered as the appropriate tensors. These results are also in accordance with the input parameters introduced for obtaining the right solutions (minor agglomeration pattern), which included a relatively high value for the shear stress ratio (Fig. 6).

The significance of both types of solutions obtained by cluster analysis can also be easily understood with the aid of the graphical representation of the possible solutions depicted by the y–R diagram (Fig. 7). The y–R diagram method is a simplified fault inversion method that requires one axis of the stress tensors to be vertical or nearly vertical, as is the case here. According to Simón-Gómez (1986), possible solutions in this diagram are those where different groups of curves produce clouds of their mutual intersections, so that five solutions (labelled from 1 to 5) could be differentiated in the Ogmore 4 site (see Fig. 7). However, all of these solutions are not possible because each fault-slip (curve) or group of fault-slips (group of curves) can be explained by just one stress tensor (Simón-Gómez, 1986; Liesa, 2000), so some solutions are mutually incompatible (e.g. solutions 1 and 4 or 2 and 3). Solutions 1 and 2 are in agreement with tensors 1 and 2, respectively, while solutions 3 and 4 are in accordance, respectively, with tensors 3 and 4 (Table 4).

If the Andersonian geometry of this dataset described by Lisle and Vandycke (1996), defined by dextral faults trending 020° and sinistral faults trending 050°, is now considered the correct tensors can be constrained with confidence. So, an Andersonian model of strike-slip faults, which is depicted on the y–R diagram as tensor 2 (compression roughly 030°), represents the appropriate tensor. According to the y–R diagram, solutions and therefore tensors 3 and 4 are inappropriate tensors, leaving only 1 and 5 as other appropriate tensors. The stress tensor of solution 5 has the principal stress axes in the same positions as tensor 2 but has a higher stress ratio (Fig. 7). The faults that define solution 5 in the y–R diagram have, however, also been included in tensor 2, and they are located essentially on the right part of the Mohr circle (Fig. 6). This location is explained because these faults represent the reactivation, with opposite senses of movement, of fault planes formed during the compression 110°. So, it is likely that 090°–105°-trending, dextral strike-slip faults formed during the compression 110°, were later reactivated as sinistral during the compression 030°, and formerly 120°–140°-trending, sinistral strike-slip faults were reactivated as dextral (Fig. 7). The chronological relationship between the dextral (fault 39) and sinistral (fault 40) movements on a fault plane striking 105° (Fig. 7) agrees with this interpretation because tensor 1 explained the dextral movement and tensor 2 the sinistral movement.

Number of test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Grid spacing (°)	15	15	15	15	15	22	22	22	22	22	30	30	30	30	30	30	30	30
Misfit threshold (°)	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	20	30	30
Shear criterion (τ/σ)	NO	0.47	0.58	0.7	0.84	NO	0.47	0.58	0.7	0.84	NO	0,47	0.58	0,7	0,84	NO	NO	0,58
TYPE OF RESULT	a	a	a	a-b	a-b	a	a	a	a	a	a	a	b	b	b	a	a	a



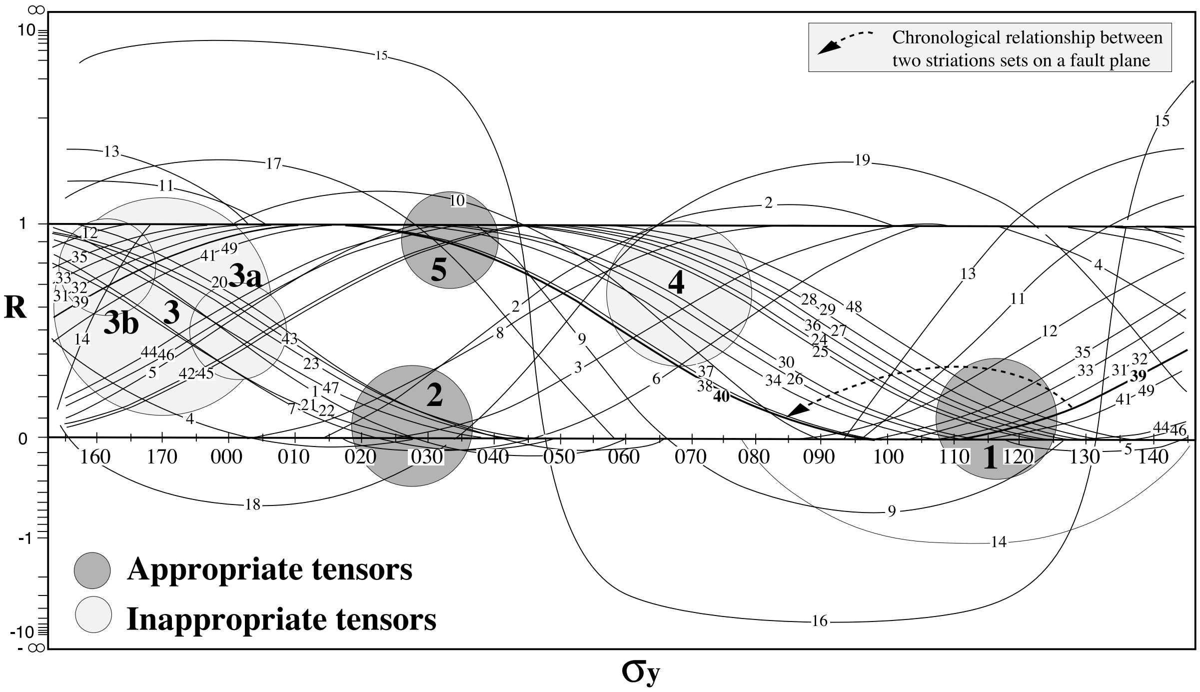


Fig. 7. Results of the y–R diagram (Simón-Gómez, 1986) for fault-slip data at Ogmore 4 site. See explanation on the text.

In conclusion, the field data allow us to discriminate between appropriate and inappropriate tensors and support the existence of two different stress tensors with a strike-slip configuration. Firstly, one compression ESE–WNW (110°) created two sets of faults that describe an Andersonian system: dextral strike-slip faults trending 090°–105° and sinistral strike-slip trending 120°–140°. After, a compression directed towards NNE–SSW (035°) produced a new Andersonian system of faults with dextral faults trending 020° and sinistral faults trending 050° and caused slip on faults of the formerly Andersonian system that were reactivated with opposite senses of slip.

This exhaustive study has led to an interpretation of the Ogmore site 4 quite different to that proposed by Lisle and Vandycke (1996). By using just the cluster analysis procedure, these authors separated two strike-slip tensors (see Fig. 4) very close to tensors 3 and 4 shown here, but significantly different to appropriate tensors (tensors 1 and 2) separated in this research. Tensor 1 (σ_1 trending 111°, $\Phi = 0.24$) is completely different to the comparable tensor with σ_1 trending 179° ($\Phi = 0.34$) inferred by Lisle and Vandycke (1996). The tensor 1 moreover is in accordance with the SP1 stylolite peaks (Fig. 4) of Lisle and Vandycke (1996) that also suggest a σ_1 in an E–W horizontal direction. This ESE–WNW compression that was supposed to be pre-Triassic in age by these authors should be reconsidered because it clearly affects Triassic rocks. Tensor 2 determined here in the Triassic rocks with σ_1 aligned in a NNE trend is closer to tensors of similar σ_1 trend inferred in the Carboniferous Limestone by Lisle and Vandycke (1996) than the tensor determined by these authors, so that this stress tensor is now defined with more confidence.

5. Discussion and conclusions

The results presented here suggest that all investigated methods have some limitations for separating stress tensors from a heterogeneous data set. These limitations are different for each method and depend greatly of the nature of the heterogeneous data. Problems for differentiating tensors generally increase when the homogeneous groups belonging to each tensor are very different in number of fault-slip data or when the tensors are closer in the orientation of stress axes.

In the cluster analysis approach, different input parameters selected by the user (grid spacing, misfit threshold, and shear stress ratio) lead sometimes to different clustering of faults. Frequently differences only appear in the level of clustering lower than the main subgroups of faults, i.e. the number of main clusters that can still be distinguished, but occasionally major differences in clustering of faults leads to grouping faults belonging to different subgroups. In this

Fig. 6. The major (a) and minor (b) agglomeration patterns obtained by cluster procedure from the Ogmore 4 site fault-slip data. The table shows the input parameters used for the different tests and the two main clustering patterns obtained in the analysis. (a) and (b) Each represents an example of the agglomeration pattern and inferred stress tensors for the major (test 1) and minor (test 13) agglomeration pattern.

sense, we propose the combined use of the cluster procedure and y–R diagram method, when this method can be applied, in order to distinguish appropriate and inappropriate stress tensors. This is based on several reasons: (1) the y–R diagram allows us a visual analysis of the grouping process of the cluster results which would be useful for the correct interpretation, (2) the initial clustering of faults in the dendrogram corresponds to the grouping of curves in the y–R diagram (see Figs. 6b and 7), and (3) both methods allow a fast identification of faults by their numbers.

Based on our study, the most sensitive parameter producing significant differences in the results is the misfit threshold angle between real and theoretical striae. We believe that the input value should be in agreement with the angular misfit theoretical striae/real striae that usually are considered as appropriate in the inversion processes. A great variety of values have been proposed as threshold values, such as 20° (Etchecopar et al., 1981) or 15° (Angelier, 1979a), but according to the exhaustive studies carried out by Casas (1990) and Casas et al. (1990) a smaller value (12° or 10°) should be adopted. Because the similarities are calculated between pairs of faults and cluster analysis considers more faults, a more restrictive condition should be accounted for. Our tests with cluster analysis suggest that when its value is high enough (15° or greater) a major misgrouping of faults is produced and when its value is small (6° or less) the fragmentation of the dendrogram is very noticeable, so that its interpretation is quite difficult. Accordingly, we propose the extensive use in cluster analysis of 10° as the input value of the misfit threshold.

Another important parameter in cluster analysis is the shear stress ratio. Depending on this ratio, the cluster procedure tends to group faults that can be preferentially explained as newly formed faults (when a higher shear stress ratio is input) or as reactivated faults (when this parameter is not considered or its value is low), so that different solutions could occur. Different grid spacing also leads sometimes to different agglomeration patterns in cluster procedure. Because of this we propose the use of several different combinations in input looking for a wide range of possible solutions. After the inversion of these solutions the stress tensors should be evaluated and compared with results of other inversion methods or with other types of structural data.

The multiple inverse method (Yamaji, 2000) achieves good solutions under general conditions but it has some limits of application when the number of faults of the individual subgroups are quite different and also when both individual tensors are similar in the stress state configuration. As has been demonstrated previously, if the number of faults in each subgroup is quite different the procedure highly decreases the probability of appearance of combinations of the minor subgroups and then minor stress tensors are very difficult to detect. If both expected tensors have strike-slip configuration, the method is not able to separate the tensors even when the horizontal angular difference between both σ_1 was 60° (case 3). This is also a consequence of the fault combination procedure that leads to spurious solutions halfway between both original stress tensors. So, these drawbacks are intrinsic to the method that introduces some artefacts based on the grouping of the fault-slip data in subset k-faults. They will probably be common to other methods that use similar procedures, that is, those methods that analyse subsets of faults by inversion procedures and use the clustering of these solutions in order to define stress tensors and therefore homogeneous fault-slip data. Other disadvantages of this method arise because the results do not permit the identification of the faults defining the stress tensors and therefore additional field information, such as that related to the chronological relationship between different striations on a fault plane, cannot be used to relatively date the inferred stress tensors.

The Etchecopar method (Etchecopar et al., 1981), used in combination with the y–R diagram method (Simón-Gómez, 1986), usually gives good results but generally some faults are wrongly assigned to subgroups. In any case, these faults are normally so few that they do not affect greatly the determined stress tensor. However, when one of the tensors consists of a small number of faults (case 2), there is a great variability in the determined stress tensors because the misgrouping faults can modify the results. If the quality criteria provided by this method (e.g. the Mohr circle) or the results of additional methods such as the y–R diagram are taken into account these rogue faults can usually be identified and removed from the solution.

In general, additional field data are of great importance in the interpretation of the results obtained from fault-slip data inversion. The geometric relationships between fault planes or fault sets (e.g. Andersonian models of faults), the cross-cutting relationships between different striations on a fault plane, and/or additional information of other structures such as tectonic stylolites and tension gashes, allow the researcher to make, in some cases, the correct choice of stress tensors. The choice of fault-slip criteria affects which faults are included in the eventual solution. In our tests (two homogeneous data sets merged together) or in the analysis of Ogmore 4 site data, for example, the fact that most of the faults represent newly formed planes (as can be inferred from their Andersonian quasiconjugate configurations) has been used to select the appropriate stress tensors. In such tensors, the fault planes are favourably oriented for slip (located in the left-upper part of the Mohr circle), so that they are mechanically consistent with the data. The study of the case where both homogeneous fault sets represent reactivated faults has not been carried out here and it should be the focus of further research.

Finally, our study shows that all investigated methods can yield different solutions from appropriate ones if special initial conditions are taken into account. In any case, all methods require different input data, so that it is necessary for researchers to make decisions. Our findings suggest that other data, such as geometrical data among faults or data from other structures (stylolites or tension gashes for example), must be taken into account when introducing the input conditions and/or interpreting the results in order to reach true solutions. Thus, at the moment there exists no fully automatic procedure for separating stress tensors and we believe that the attempt to propose such a method will be not successful because too many factors, such as if the faults are newly formed of reactivated fractures, are combined in the fracturing process.

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Appendix A

The modified version of the cluster program used in this study, which resulted as a consequence of this research, differs from that proposed by Nemcok and Lisle (1995) just in regard to the positions of σ_1 and σ_2 stress axes in the searching grid, which have here interchanged positions. In the new version, to obtain a set of variable tensor orientations, σ_2 orientations instead of σ_1 orientations were chosen according to the spherical grid pattern as is explained by Nemcok and Lisle (1995). For each selected σ_2 orientation a variety of σ_3 directions is produced by incremental rotations through an angle β (grid spacing angle) about the σ_2-axis. Afterwards, taking into account the positions of σ_2 and σ_3 stress axes the position of σ_1 is calculated from the orthogonality constraint. This modification means that σ_1 and σ_3 have the same number of different positions and spatial distribution in the searching grid, which therefore leads to a more homogeneous and efficient search for the compatibility between pairs of faults.

Moreover the grid spacing angle, the misfit threshold angle and the shear stress condition are the other input parameters used by the cluster procedure. The first one considers the difference angle between the striation pitch predicted using Bott's (1959) equation for each tensor and the measured striation pitch on the fault. If the discrepancy exceeds the threshold angle, the fault and tensor are considered incompatible. The shear stress condition may also be introduced to consider whether or not a fault fits each tensor. When a value of the shear stress/normal stress (τ/σ) ratio is introduced, the faults that fit each tensor must also possess a higher value than the input value.

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